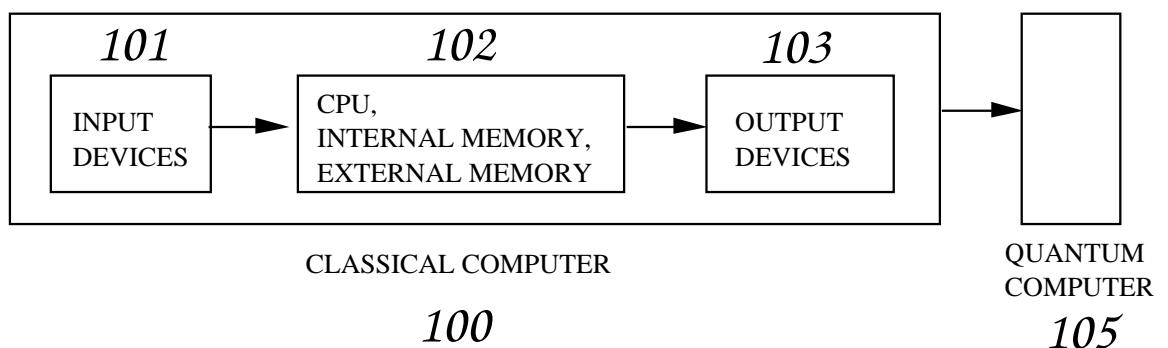
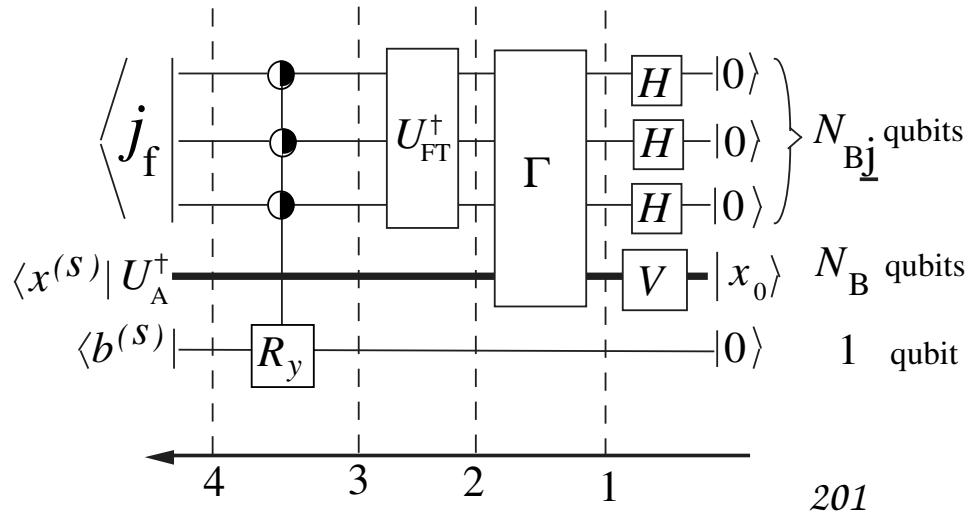


*Figure 1*



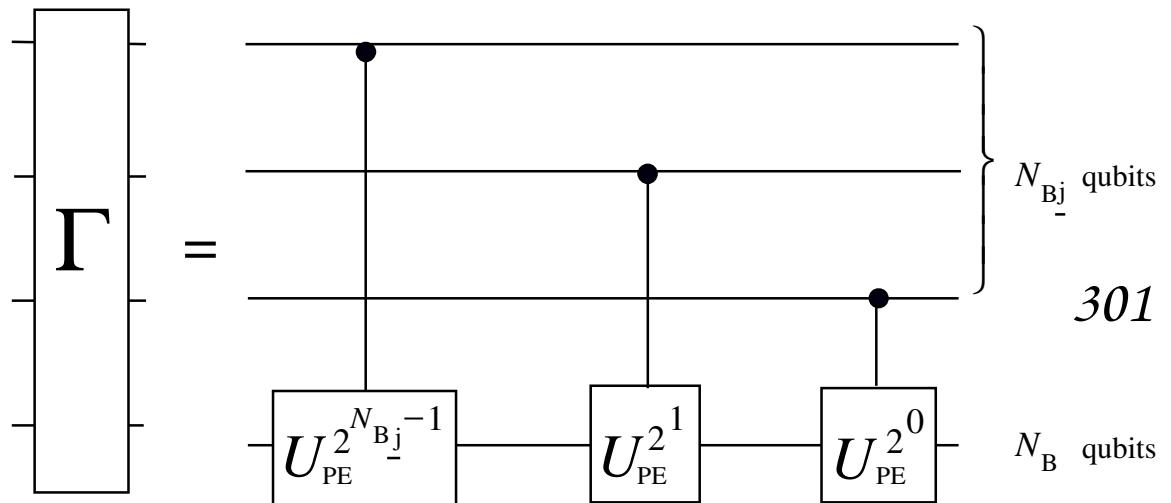
*Figure 2*



$$\mu(x_0) \stackrel{\text{def}}{=} \langle x_0 | V^\dagger f(A) V | x_0 \rangle \quad 202$$

$$\mu(x_0) = \frac{1}{\gamma N_{sam}} \sum_{s=1}^{N_{sam}} \delta_0^{b^{(s)}} \quad 203$$

*Figure 3*

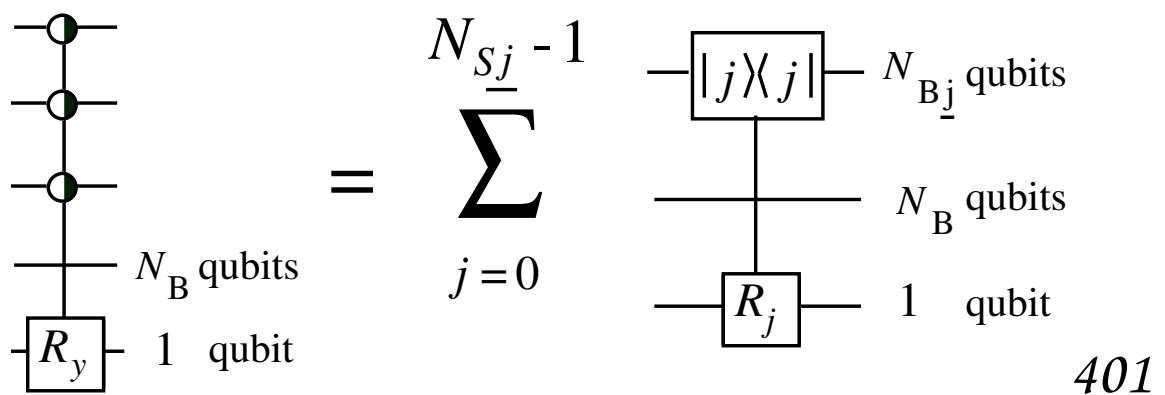


$$= \sum_{j=0}^{N_{Sj}-1} -\boxed{|j\rangle\langle j|} - N_{Bj} \text{ qubits} \quad 302$$

$-\boxed{U_{\text{PE}}^j} - N_B \text{ qubits}$

$$U_{\text{PE}} = e^{i A \Delta t} \quad 303$$

*Figure 4*



$$R_j = \begin{bmatrix} c_j & -s_j \\ s_j & c_j \end{bmatrix} \quad 402$$

$$c_j = \sqrt{\gamma_f \left( \frac{2\pi j}{\Delta t N_{S\underline{j}}} \right)} \quad 403$$

$$s_j = \sqrt{1 - c_j^2} \quad 404$$

*Figure 5*

*Figure 6*

$$\langle y | U_\Omega | x \rangle = \langle y | \hat{\Omega} = \Omega_x \rangle, \quad Z = \text{tr}(e^{-\beta H}) \quad 601$$

(a)

$$V = U_\Omega, \quad A = \rho, \quad f(\xi) = \xi \quad 602$$

$$\mu(x) = \langle x | U_\Omega^\dagger \rho U_\Omega | x \rangle \quad 603$$

$$\text{tr}(\Omega \rho) = \frac{1}{N_{sam}} \sum_{s=1}^{N_{sam}} \Omega_{x^{(s)}} \quad 604$$

(b)

$$V = U_\Omega, \quad A = H, \quad f(\xi) = e^{-\beta \xi} \quad 605$$

$$\mu(x) = \langle x | U_\Omega^\dagger e^{-\beta H} U_\Omega | x \rangle \quad 606$$

$$\text{tr}(\Omega \rho) = \frac{1}{N_{sam}} \sum_{s=1}^{N_{sam}} \Omega_{x^{(s)}} , \text{ where } \rho = \frac{e^{-\beta H}}{Z} \quad 607$$

(c)

$$V = 1, \quad A = H, \quad f(\xi) = e^{-\beta \xi} \quad 608$$

$$\mu(x) = \langle x | e^{-\beta H} | x \rangle \quad 609$$

$$\check{P}(x) = \frac{1}{N_{sam}} \sum_{s=1}^{N_{sam}} \delta_{x^{(s)}}, \quad \check{Z} = \frac{1}{N_{sam}} \sum_{s=1}^{N_{sam}} \frac{\mu(x^{(s)})}{\check{P}(x^{(s)})} \approx Z \quad 610 \quad 611$$