# Group Theoretic Bayesian Networks

Robert R. Tucci
P.O. Box 226
Bedford, MA 01730
tucci@ar-tiste.com

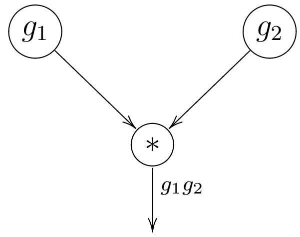
March 16, 2015

This short article was written as an appendix to a post entitled "Bayesian Networks Do Groups Too" for my blog

"Quantum Bayesian Networks" (www.qbnets.wordpress.com)

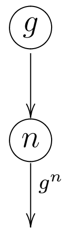
Suppose $\mathcal{G}$ is a group. Let $\delta(a, b)$ equal 1 if $a = b$ and zero otherwise.

**Group multiplication:** For any $g, g_1, g_2 \in \mathcal{G}$, let $P(g|g_1, g_2) = \delta(g, g_1 g_2)$ be the transition matrix for the node with an asterisk surrounded by a circle:




(1)

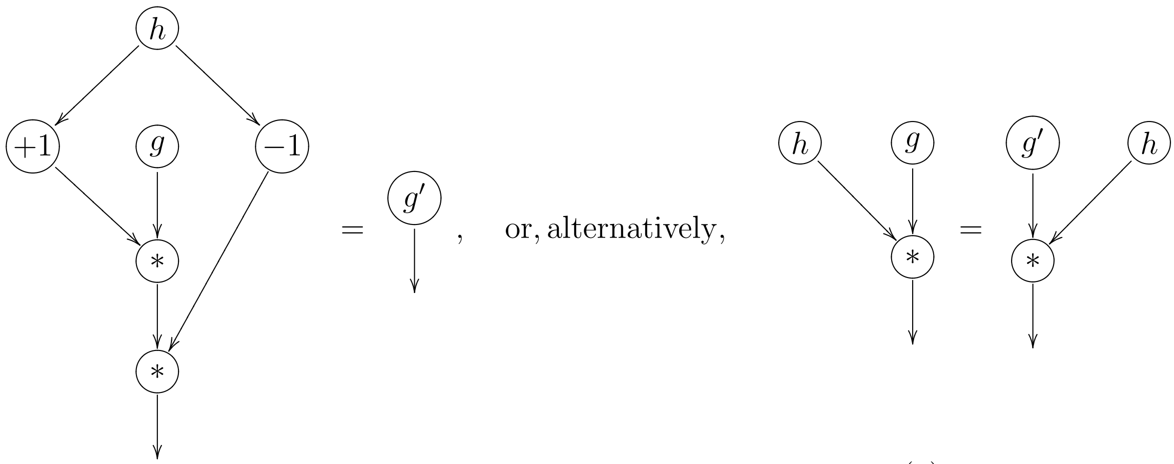
**Group n'th power:** Suppose $n$ is any integer, possibly zero or negative. For $g', g \in \mathcal{G}$, let $P(g'|g) = \delta(g', g^n)$ be the transition matrix for the node with $n$ surrounded by a circle:




(2)

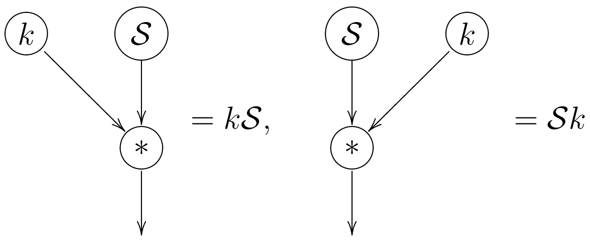
When $n = -1$, this node returns $g^{-1}$, the inverse of $g$, and when $n = 0$, it returns $g^0 = e$, the identity element of the group.

**Conjugate elements:** $g, g' \in \mathcal{G}$ are said to be a conjugate pair of elements of $\mathcal{G}$ if there exists $h \in \mathcal{G}$ such that




(3)

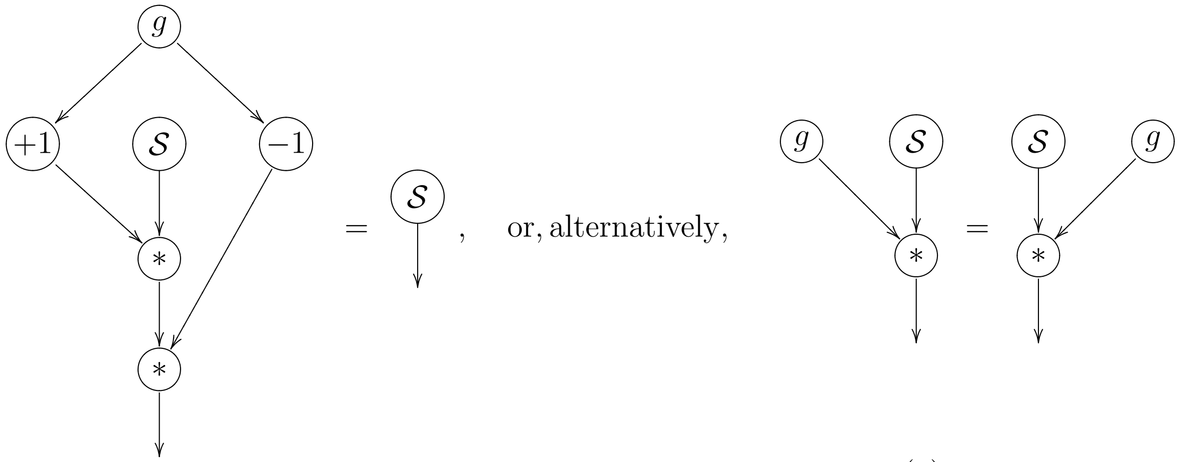
**Right and Left cosets:** Suppose $\mathcal{S}$ is a subgroup of $\mathcal{G}$. For any $k \in \mathcal{G}$, define




(4)

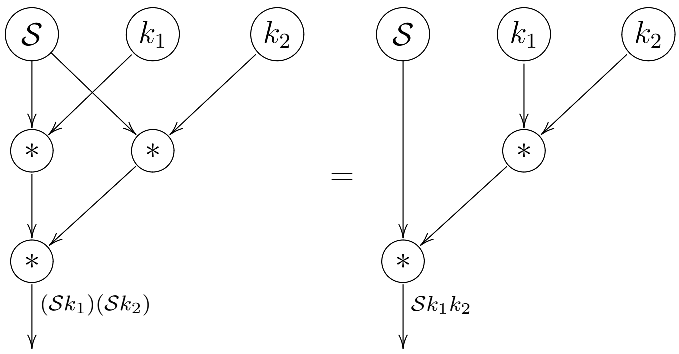
**Invariant or Normal subgroup:** $\mathcal{S}$ is an invariant or normal subgroup of

$\mathcal{G}$, if it is a subgroup of $\mathcal{G}$ such that, for all $g \in \mathcal{G}$,




(5)

**Coset multiplication:** Suppose $\mathcal{S}$ is an invariant subgroup of $\mathcal{G}$. For any $k_1, k_2 \in \mathcal{G}$,




(6)

One defines $\frac{\mathcal{G}}{\mathcal{S}} = \{\mathcal{S}k : k \in \mathcal{G}\}$. Then $\mathcal{G} = \mathcal{S} \times \frac{\mathcal{G}}{\mathcal{S}}$.